Human-Horse Interaction Model in Jumping

鶴尾明大[1] リングホーファー萌奈美[2] 山本真也[2] 池田和司[1]
Asahi Tsuruo[1], Monamie Ringhofer[2], Shinya Yamamoto[2], Kazushi Ikeda[1]

[1] 奈良先端科学技術大学院大学
[1] Nara Institute of Science and Technology
[2] 京都大学
[2] Kyoto University

Abstract: In horse-riding, a rider controls the horse through their interaction so that the horse runs fast or jumps over hurdles. Even though the rider has an important role in controlling a jumping horse, the rider's policy is still open in the jumping. Thus, to identify the rider's policy in the jumping as the understandable quality is required. As the quality, we applied the peak force, the peak power, and the total work that the horse needs to produce. To identify the rider's policy in the interaction, we regarded a jumping horse and its rider as the one-dimensional system in the vertical displacement and modeled it, based on spring-damper-mass models proposed in horse's trot, using the displacements of the centers of gravity of horses and riders from video data. The result indicates that the rider's policy is to minimize the peak force and power that the horse produces at landing. This means that the rider's policy can be to suppress the sudden muscle motion of the horse at landing in the jumping. In addition, the nature of horse-rider interaction in the jumping would be the mass-point bouncing in the vertical displacement. In conclusion, this study provides insight into the nature of horse-rider interaction in the jumping and the rider's policy.

1 Introduction

In horse-riding, a rider controls the horse through their interaction so that the horse runs fast or jumps over hurdles. Moreover, the rider's proficiency would have effects on the horse in the interaction [1, 2]. To analyze such an effect, the quantification of the movement is essential. To quantify the movement, a model representing the main features of the phenomena is required [3–5]. For this purpose, three spring(-damper)-mass (SDM) models were proposed in horse trot [6]. The models regard the horse-riding as the one-dimensional system in the vertical displacement. Two of them (SDM with a forcing function for the rider and SDM with an active spring for the rider) modeled the vertical displacements of a horse and its rider. Moreover, the models showed a decrease in the horse's work in different riding techniques in the trot. The trot is quantitatively understood, being modeled. In contrast, the quantitative understanding of the riding in horse jumping is still open.

In horse jumping, the quantitative understanding of the riding is still insufficient due to a lack of effective models although the riding in the jumping has been investigated in various aspects [7–12]. Hence, the rider's policy is unclear on the effects in the horse-rider interaction even though the rider has an important role in controlling the horse [12–14]. Thus, an effective model is necessary to understand the rider's policy in the interaction in the jumping. Here, one of the successful models, SDM with an active spring for the rider (Fig. 1), has the applicability for the jumping because the model has two plausible factors for the jumping. The first is force contact factors that can represent contact and suspension phases of a horse and its rider. The second is an active spring for the rider that can control the rider's movement. Therefore, the models would be capable of representing a jumping horse and its rider in the interaction although it is needed to modify the horse's model.

The purpose of this study is to identify the rider's policy as the understandable quality in order to understand their movement and its proficiency in the horse-rider interaction in the jumping. We hypothesized that the rider's policy is the minimization of mechanical properties that the horse needs to produce. As the mechanical properties, the peak force, the peak power, and the total work were applied. To this end, we proposed a model of a jumping horse and its rider as the one-dimensional system in the vertical displacement, based on SDM with an active spring of the rider, and confirmed the rider's policy in the interaction on the simulational approach using the proposed model.

2 Materials and Methods

2.1 Mathematical model of a jumping horse and its Rider

The proposed model of a jumping horse and its rider is the same with SDM with an active spring of the rider (Fig. 1) except for the mechanism of the horse's ver-

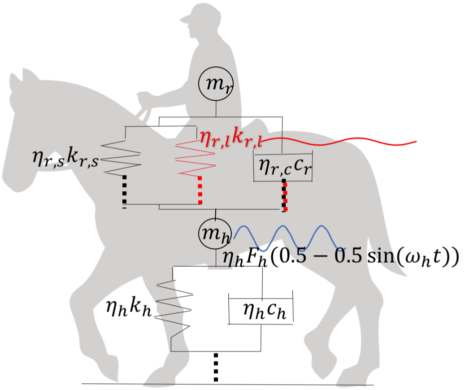


Figure 1: Spring-Damper-Mass with an active spring for the rider. $k_{r,s}$: rider's constant spring stiffness, $k_{r,l}$: rider's variable spring stiffness, k_h: horse's spring stiffness, c_r: rider's damping coefficient, c_h: horse's damping coefficient, m_r:rider's mass, m_h: horse's mass, F_h: horse's amplitude of the forcing function, ω_h: horse's angular frequency of the forcing function, $\eta_{r,s}$: rider's force contact factor for $k_{r,s}$, $\eta_{r,l}$: rider's force contact factor for $k_{r,l}$, η_h: horse's force contact factor, t: time.

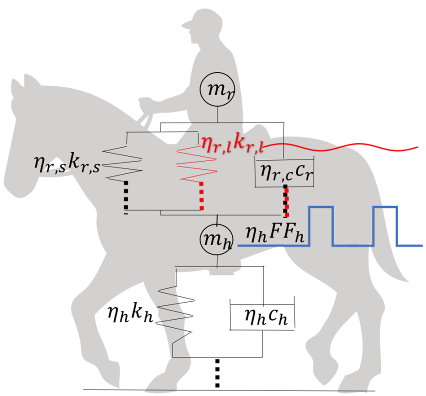


Figure 2: Spring-Damper-Mass with an active spring for the rider and a square wave forcing function for the horse. $k_{r,s}$: rider's constant spring stiffness, $k_{r,l}$: rider's variable spring stiffness, k_h: horse's spring stiffness, c_r: rider's damping coefficient, c_h: horse's damping coefficient, m_r: rider's mass, m_h: horse's mass, FF_h: horse's square wave the forcing function, $\eta_{r,s}$: rider's force contact factor for $k_{r,s}$, $\eta_{r,l}$: rider's force contact factor for $k_{r,l}$, η_h: horse's force contact factor, t: time.

tical oscillations. In the model, the bodies of a horse and its rider are represented by mass points m_h and m_r. Each is connected with springs and damper(s) and has contact factors (Fig. 2).

2.1.1 Rider's model

SDM with an active spring for the rider has two springs with constant stiffness $k_{r,s}$ called a saddle spring and variable stiffness $k_{r,l}$ called an (leg) active spring. The model is originally for the trot. In the rising trot, which is a kind of riding techniques, the rider has the standing and sitting phases. In the sitting phase, the rider's biomechanical properties are determined by a saddle, the upper body connected with the saddle, and the legs. In contrast, in the standing phase, the property is mainly determined by the legs that will change the effective stiffness because of losing connection between the saddle and the upper body. For this reason, the two springs were introduced in the model.

The stiffness $k_{r,l}$ takes a sinusoidal value from $k_{r,l,base}$ to $k_{r,l,base} + k_{r,l,amp}$ with phase difference γ_r and the angular frequency ω_r ($2\pi f_r$, where f_r is the time frequency). In addition, the two springs have the force contact factors $\eta_{r,s}$ and $\eta_{r,l}$, respectively. In total, the dynamics of the rider at time t is described as

$$\begin{aligned}
m_r\ddot{z}_r &= -\eta_{r,c}c_r(\dot{z}_r - \dot{z}_h) \\
&\quad - \eta_{r,s}k_{r,s}\varepsilon_{r,s} - \eta_{r,l}k_{r,l}\varepsilon_{r,l} - m_r g, && \text{(1a)}\\
k_{r,l} &= k_{r,l,base} \\
&\quad + k_{r,l,amp}(0.5 - 0.5\sin(\gamma_r + \omega_r t)), && \text{(1b)}\\
z_{r,\eta_l} &= z_{r,\eta_l,base} - z_{r,\eta_l,amp}\sin(\gamma_r + \omega_r t), && \text{(1c)}\\
\varepsilon_{r,s} &= \frac{(z_r - z_h) - z_{r,\eta_s}}{z_{r,\eta_s}}, && \text{(1d)}\\
\varepsilon_{r,l} &= \frac{(z_r - z_h) - z_{r,\eta_l}}{z_{r,\eta_l}}, && \text{(1e)}\\
\eta_{r,s} &= \frac{1}{1+\exp(a\varepsilon_{r,s})}, && \text{(1f)}\\
\eta_{r,l} &= \frac{1}{1+\exp(a\varepsilon_{r,l})}, && \text{(1g)}\\
\eta_{r,c} &= \begin{cases} \eta_{r,s} & (\eta_{r,s} \geq \eta_{r,l}), \\ \eta_{r,l} & (\eta_{r,s} < \eta_{r,l}), \end{cases} && \text{(1h)}
\end{aligned}$$

where z is the vertical displacement, $\dot{z}$ is the vertical velocity, ε is the strain of the movement, η is the force contact factor, z_{r,η_s} and $z_{r,\eta_l,base}$ are the average of the difference between the heights of the rider and the horse, g is the constant gravitational acceleration, and a is a constant that determines the tendency of the contact's switch.

2.1.2 Horse's model

SDM with an active spring for the rider is originally for horse's trot. Thus, the model assumed that the vertical oscillations of the horse's body are excited by a motor system that is described as a sine wave forcing function (Fig. 1). Meanwhile, the jumping is not like the motor system. Accordingly, we equipped the model with a square wave forcing function FF_h for the jumping horse instead of the sine wave forcing function (Fig. 2).

The dynamics of the horse at time t is described as

$$\begin{aligned} m_h\ddot{z}_h &= -\eta_h c_h \dot{z}_h - \eta_{r,s} c_r(\dot{z}_h - \dot{z}_r) - \eta_h k_h \varepsilon_h \\ &\quad + \eta_{r,s} k_{r,s}\varepsilon_{r,s} + \eta_{r,l} k_{r,l}\varepsilon_{r,l} \\ &\quad - m_h g + \eta_h FF_h, \end{aligned} \tag{2a}$$

$$\varepsilon_h = \frac{z_h - z_{h,\eta}}{z_{h,\eta}} \tag{2b}$$

$$\eta_h = \frac{1}{1+\exp(a\varepsilon_h)}, \tag{2c}$$

$$FF_h = \begin{cases} A_{h\ takeoff} & (t_{s1} \le t \le t_{e1}) \\ A_{h\ landing} & (t_{s2} \le t \le t_{e2}) \\ 0 & (otherwise) \end{cases} \tag{2d}$$

where $A_{h\ takeoff}$ is the amplitude for the take-off from time t_{s1} to t_{e1} and $A_{h\ landing}$ is the amplitude for the landing from time t_{s1} to t_{e1}. $A_{h\ landing}$ was introduced since the horse decelerates downwards by itself at landing in terms of vertical CoG's displacement.

2.2 Data acquisition for parameter estimation

The proposed model is a mass-point model in the vertical displacement. The centers of gravity (CoGs) in the vertical displacement are needed to build the models. For this, first, we got anatomical positions of riders and horses from video data and calculated the CoGs using the positions extracted.

Two videos for the jumping (Data1[1]: fps 29.7 and Data2[2]: fps 25) were collected from the Internet videos site. From the two videos, anatomical positions of the riders and the horses were extracted by using DeepLabCut (a tool for markerless pose estimation of body parts based on deep learning) [15]. The scales (pixel/m) of the videos were calibrated so that the horse withers height was 1.6 m.

The anatomical positions for a rider were the head, the shoulders, the hip, the elbows, the wrists, the fingertips, the knees, the ankles, the heels, and the toes. The extracted points with low likelihood for estimation and those that appear wrong were manually corrected. The extracted points were used to calculate the CoGs of each of the four body parts (the upper body, the upper legs, the lower legs, and the feet) as done in [16] and the CoG of the rider using

$$z_{\mathrm{G}} = \sum_{i=1}^{n} \frac{z_{\mathrm{G},i} m_i}{m}, \tag{3}$$

where m is the mass, $z_{\mathrm{G},i}$ and m_i are the CoG position and the mass in each body parts, respectively. The mass of the rider was set to 60 kg.

The anatomical positions for a horse were the nine points and were used to calculate the CoG of each of the five body parts (the head, the neck, the trunk, the shoulders, and the thighs) as done in [17] and the CoG of the horse using (3). Note that we used more parts to calculate the CoG of the horse than the previous study [6] where the CoG was the average of the spinous processes of the sixth thoracic and the firt lumbar vertebrae, because the horse moves its neck to maintain the body balance during jumping [7]. The mass of the horse was set to 600 kg.

The CoGs were calculated frame by frame and then filtered by Savitzky-Golay filter [18] for smoothing.

2.3 Estimation of model parameters

The parameters in the model were estimated from the observed displacements z_r and z_h by using DE [19,20]. Here, the objective function for the estimation was the mean square error from the observed and modeled displacements of the horse's and rider's CoG. The search range of the parameters are presented in Table. 1, which was determined from the values in [6, 21–23] and in this study. To evaluate the error between the observed and modeled displacements, the differential equations in the models were solved in RK45 (scipy.integrate.solve_ivp [24]) during parameters' estimation in DE.

2.4 Rider's policy in the horse-rider interaction

The mechanical values were calculated in order to show that the rider's policy is to minimize the mechanical values that the horse produces using SDM with an active spring for the rider and a square wave forcing function for the horse.

As the understandable quality for the rider's policy, the force $\mathbf{F}_{h,g}$, the absolute power P_h , and the total work W_h that the horse produces to the ground were calculated as

$$\mathbf{F}_{h,g} = -\eta_h c_h \dot{z} - \eta_h k_h \varepsilon_h + \eta_h FF_h \tag{4}$$

$$P_h = \mathbf{F}_{h,g}|\dot{z}_h| \tag{5}$$

$$W_h = \int P_h dt \tag{6}$$

We calculated peak forces, absolute peak powers, and total works that the horse produced to the ground using Equations (4)–(6) during take-off and landing. The mechanical values were calculated at the estimated parameters of the rider and at the perturbed parameters of the rider. The perturbed parameters of the rider are shifted parameters of the estimated ones, where active spring base stiffness $k_{r,l,base}$ (kN m^{-1}) and damping coefficient c_r (kg s^{-1}) were

[1] `https://www.youtube.com/watch?v=GS8WGSPZAKU`
[2] `https://www.youtube.com/watch?v=Gtn2W8-QbjI`

Table 1: Search range of the model parameters for the horse and the rider

Horse's parameters	Search range
damping coefficient c_h (kg s^{-1})	0 – 10000
spring stiffness k_h (kN m^{-1})	0 – 80
amplitude $A_{h\ takeoff,\ landing}$ (N)	0 – 12000
take-off time window t_{s1}, t_{e1} (s)	0 – half of the whole time
landing time window t_{s2}, t_{e2} (s)	half of the whole time – end of the time
Rider's parameters	**Search range**
damping coefficient c_r (kg s^{-1})	0 – 3000
saddle spring stiffness $k_{r,s}$(kN m^{-1})	0 – 80
active spring base stiffness $k_{r,l,base}$ (kN m^{-1})	0 – 40
active spring increase stiffness $k_{r,l,amp}$ (kN m^{-1})	0 – 40
phase difference γ_r	0 – 2π
leg's amplitude $z_{r,\eta_l,amp}$ (m)	0 – 0.3
time frequency f_r (Hz)	0 – 3

shifted. Note that the shifts are of the rider's parameters, not of the horse's ones, and $\dot{z}_h$ and ε_h in Equations (4)–(6) were calculated in the estimated parameters and in each set of the perturbed parameters.

The calculation of the mechanical values were conducted in order to show that the rider's policy is to minimize the mechanical values that the horse produces. Hence, this assumed that the mechanical values are minimized at the estimated parameters, not at the perturbed parameters.

3 Results

SDM with an active spring for the rider and a square wave forcing function for the horse successfully modeled the observed displacements of the horses and the riders (Fig. 3)(Coefficient of determination R^2 , Data 1: rider= 0.990, horse= 0.952, Data 2: rider=0.985, horse= 0.906).

The results of the calculations of the mechanical values showed that, at landing, the peak forces on Data 1 and 2 (Fig. 4 (d, j)) and the peak power on Data 1 (Fig. 4 (e)) at the estimated parameters were the smallest and that the peak power on Data 2 (Fig. 4 (k)) and the total works on Data 1 and 2 (Fig. 4 (f, l)) at the estimated parameters was smaller than ones at the perturbed parameters. In contrast, at take-of, the mechanical values did not drastically change at any points (Fig. 4 (a – c), (g – i)).

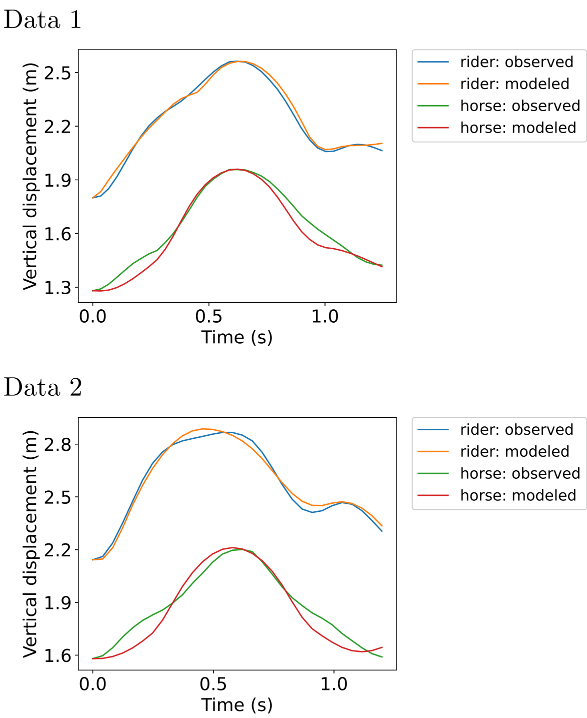


Figure 3: Vertical the displacements observed and modeled by SDM with an active spring for the rider and a square wave forcing function for the horse. Rider observed (blue line) : rider's displacements observed from video data, rider modeled (orange line) : rider's displacements modeled by the model, horse observed (green line) : horse's displacements observed from video data, horse modeled (red line) : horse's displacements modeled by the model. Coefficient of determination R^2 , Data 1: rider= 0.990, horse= 0.952, Data 2: rider=0.985, horse= 0.906

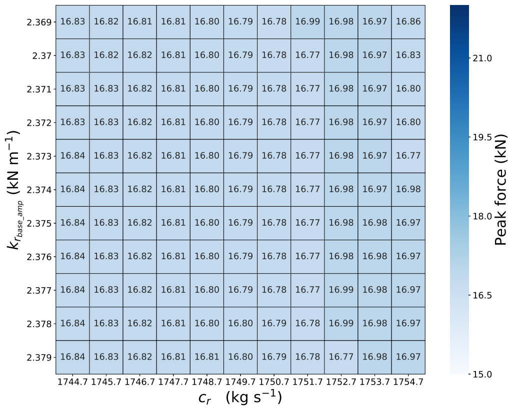
4 Discussion

To quantitatively understand the rider's policy in terms of the mechanical perspective, we modeled a jumping horse and its rider, adjusting the horse's model of SDM with an active spring for the rider. The results gave some insight into the horse-rider interaction in jumping and the rider's policy. The details are discussed below.

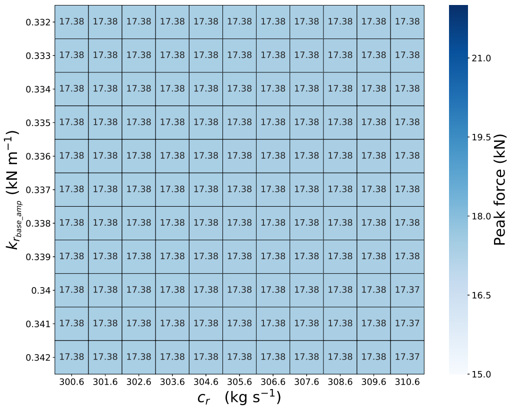
Take-off:

Data 1 | Data 2

Force

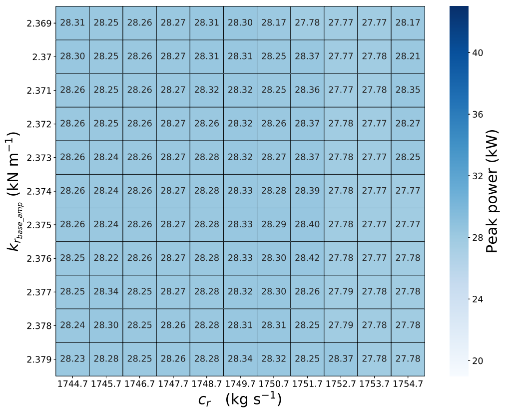


(a)

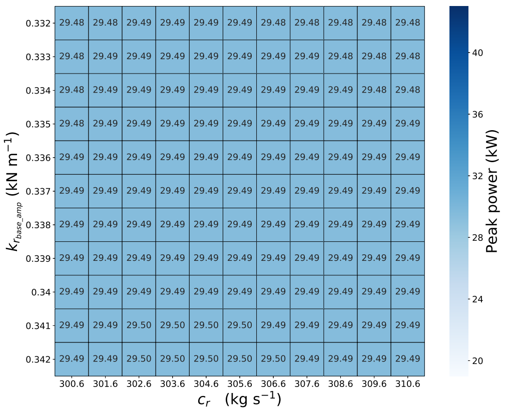


(g)

Power

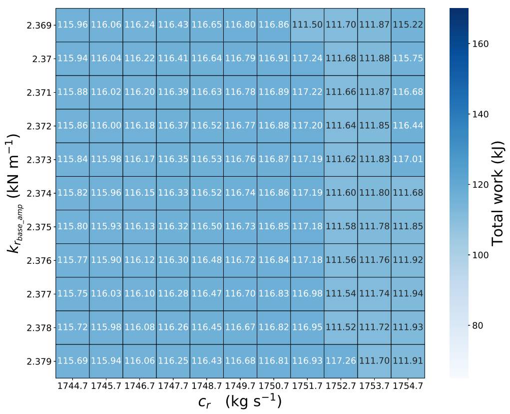


(b)

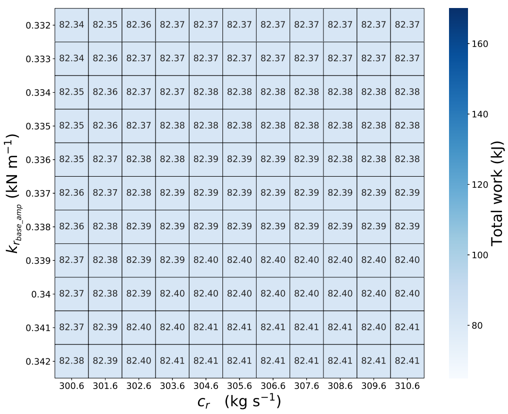


(h)

Work



(c)



(i)

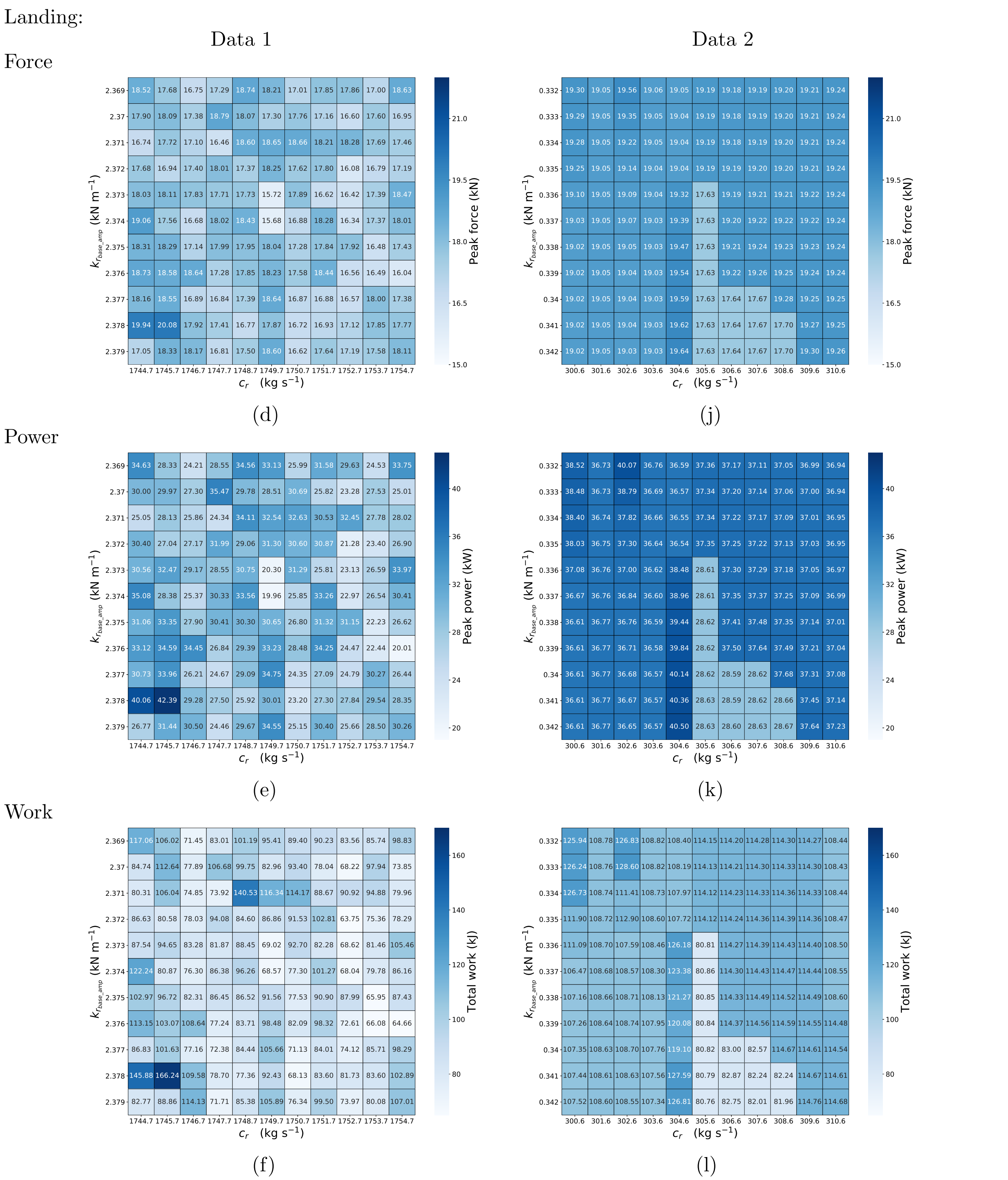


Figure 4: The mechanical values that the horse produced to the ground. (a – c): force, power, and work, at take-off on Data 1, (d – f): force, power, and work, at landing on Data 1, (g – i): force, power, and work at take-off on Data 2, (j – l): force, power, and work at landing on Data 2. Spring stiffness 0.337 kN m^{-1} and damping coefficient 305.6 kg s^{-1} are the estimated parameters of the rider on Data 1. Spring stiffness 2.374 kN m^{-1} and damping coefficient 1749.7 kg s^{-1} are the estimated parameters of the rider on Data 2. The others are the perturbed parameters of the rider.

4.1 Horse-rider interaction in the jumping

The nature of the horse-rider interaction in the jumping would be mass-point bouncing of a horse and its rider in the vertical direction, like the trot [6]. This is because the one-dimensional system of a spring-like manner successfully modeled a jumping horse and its rider (Fig. 3). Actually, all animals that run, hop, and trot bounce along the ground in a spring-like manner [25–30]. This means that the main feature of these movements is the vertical displacement in the spring like manner. Like the movements, a jumping horse and its rider that are represented by the springs' model (Fig. 3) would also essentially be the one-dimensional movement although the horse and rider are represented by the two mass-points.

4.2 Rider's policy

The results indicates that the rider's policy is to minimize the mechanical values that the horse produces at landing. In fact, the mechanical values at the estimated parameters were smaller than ones at the perturbed parameters or the smallest at landing (Fig. 4 (d – f), (i – l)). In particular, the peak force and the peak power were particularly evident compared with the work (Fig. 4 (d – f), (i – l)). This should be related to the property of the jumping. It is an "instantaneous" movement. In contrast, in the trot that is a "continuous" movement, the total work of the horse was also minimized in the optimal mode of the riding [6]. This is the difference of the property between the trot and the jumping. In the jumping, the rider's policy should be the minimization especially in the peak power and the peak force. In other words, in the instantaneous movement, that is, the jumping, the rider's policy can be to suppress the sudden muscle motion of the horse.

In addition, the results indicates that the proficiency in the rider's policy is reflected at landing, since at take-off, the mechanical values did not drastically change at any parameter points (Fig. 4 (a – c), (g – i)) while at landing, the mechanical values changed depending on the parameters. At take-off, the horse jumps up and lift up the rider under any rider's motions and positions. In contrast, at landing, the rider could control the peak of the force and the power that the horse produces, changing the rider's position and the rider's landing moment with the horse's landing moment. For these reasons, rider's proficiency could be reflected at landing.

5 Conclusion and Future Works

We found that the rider's policy would be the minimization of the mechanical values that the horse produces at landing. In particular, the peak force and the peak power were particularly evident. This is likely to be related to the property of the jumping. Besides, the horse-rider interaction in the jumping would essentially be the mass-point bouncing in the vertical displacement since a jumping horse and its rider were successfully modeled in the one-dimensional system of two springs. In conclusion, the hypothesis that the rider's policy is the minimization of the mechanical values the horse produces was confirmed.

A future work is practically to compare proficient riders with beginner's in the proposed model in order to verify our indication regarding the rider's policy that is to minimize the mechanical values at landing in more data. Proficient riders would reduce the mechanical values. This comparison will make our indication more persuasive.

References

[1] A. Münz, F. Eckardt, and K. Witte, "Horse–rider interaction in dressage riding," *Human movement science*, vol. 33, pp. 227–237, 2014.

[2] J. Lagarde, C. Peham, T. Licka, and J. S. Kelso, "Coordination dynamics of the horse-rider system," *Journal of motor behavior*, vol. 37, no. 6, pp. 418–424, 2005.

[3] J. A. Ambrósio and A. Kecskeméthy, "Multibody dynamics of biomechanical models for human motion via optimization," in *Multibody Dynamics*. Springer, 2007, pp. 245–272.

[4] C. Quental, J. Folgado, J. Ambrósio, and J. Monteiro, "A multibody biomechanical model of the upper limb including the shoulder girdle," *Multibody System Dynamics*, vol. 28, no. 1-2, pp. 83–108, 2012.

[5] M. Roller, S. Björkenstam, J. Linn, and S. Leyendecker, "Optimal control of a biomechanical multibody model for the dynamic simulation of working tasks," in *ECCOMAS Thematic Conference on Multibody Dynamics, Prague, Czech Republic, June*, 2017, pp. 19–22.

[6] P. de Cocq, M. Muller, H. M. Clayton, and J. L. Van Leeuwen, "Modelling biomechanical requirements of a rider for different horse-riding techniques at trot," *Journal of experimental biology*, vol. 216, no. 10, pp. 1850–1861, 2013.

[7] P. Nemecek, L. Cabell, and M. Janura, "Horse and rider interaction during simulated horse jumping," *Journal of Equine Veterinary Science*, vol. 70, pp. 26–31, 2018.

[8] K. Nankervis, L. Dumbell, L. Herbert, J. Winfield, R. Guire, and E. Launder, "A comparison of the position of elite and non-elite riders during competitive show jumping," *Comparative Exercise Physiology*, vol. 11, no. 2, pp. 119–125, 2015.

[9] P. N. Powers and A. M. Kavanagh, "Effect of rider experience on the jumping kinematics of

riding horses," *Equine and Comparative Exercise Physiology*, vol. 2, no. 4, pp. 263–267, 2005.

[10] M. Patterson, J. Doyle, E. Cahill, B. Caulfield, and U. M. Persson, "Quantifying show jumping horse rider expertise using imus," in *2010 Annual International Conference of the IEEE Engineering in Medicine and Biology.* IEEE, 2010, pp. 684–687.

[11] P. Galloux and E. Barrey, "Components of the total kinetic moment in jumping horses," *Equine Veterinary Journal*, vol. 29, no. S23, pp. 41–44, 1997.

[12] P. Powers and A. Harrison, "Influences of a rider on the rotation of the horse–rider system during jumping," *Equine and Comparative Exercise Physiology*, vol. 1, no. 1, pp. 33–40, 2004.

[13] D. Lewczuk, K. Słoniewski, and Z. Reklewski, "Repeatability of the horse's jumping parameters with and without the rider," *Livestock Science*, vol. 99, no. 2-3, pp. 125–130, 2006.

[14] P. Powers and A. Harrison, "Show-jumping: Effects of the rider on the linear kinematics of jumping horses," *Sports Biomechanics*, vol. 1, no. 2, pp. 135–146, 2002.

[15] A. Mathis, P. Mamidanna, K. M. Cury, T. Abe, V. N. Murthy, M. W. Mathis, and M. Bethge, "Deeplabcut: markerless pose estimation of user-defined body parts with deep learning," *Nature neuroscience*, vol. 21, no. 9, p. 1281, 2018.

[16] P. De Leva, "Adjustments to zatsiorsky-seluyanov's segment inertia parameters," *Journal of biomechanics*, vol. 29, no. 9, pp. 1223–1230, 1996.

[17] H. Buchner, H. Savelberg, H. Schamhardt, and A. Barneveld, "Inertial properties of dutch warmblood horses," *Journal of biomechanics*, vol. 30, no. 6, pp. 653–658, 1997.

[18] A. Savitzky and M. J. Golay, "Smoothing and differentiation of data by simplified least squares procedures." *Analytical chemistry*, vol. 36, no. 8, pp. 1627–1639, 1964.

[19] R. Storn and K. Price, "Differential evolution–a simple and efficient heuristic for global optimization over continuous spaces," *Journal of global optimization*, vol. 11, no. 4, pp. 341–359, 1997.

[20] S. Das and P. N. Suganthan, "Differential evolution: A survey of the state-of-the-art," *IEEE transactions on evolutionary computation*, vol. 15, no. 1, pp. 4–31, 2010.

[21] Y. Blum, S. W. Lipfert, and A. Seyfarth, "Effective leg stiffness in running," *Journal of biomechanics*, vol. 42, no. 14, pp. 2400–2405, 2009.

[22] A. A. Zadpoor and A. A. Nikooyan, "Modeling muscle activity to study the effects of footwear on the impact forces and vibrations of the human body during running," *Journal of biomechanics*, vol. 43, no. 2, pp. 186–193, 2010.

[23] M. F. Bobbert and L. J. Richard Casius, "Spring-like leg behaviour, musculoskeletal mechanics and control in maximum and submaximum height human hopping," *Philosophical Transactions of the Royal Society B: Biological Sciences*, vol. 366, no. 1570, pp. 1516–1529, 2011.

[24] "scipy.integrate.solve_ivp — scipy v1.5.4 reference guide," https://docs.scipy.org/doc/scipy/reference/generated/scipy.integrate.solve_ivp.html, (Accessed on 12/09/2020).

[25] R. J. Full and M. S. Tu, "Mechanics of six-legged runners," *Journal of experimental biology*, vol. 148, no. 1, pp. 129–146, 1990.

[26] R. Blickhan and R. J. FULL, "Locomotion energetics of the ghost crab: Ii. mechanics of the centre of mass during walking and running," *Journal of Experimental Biology*, vol. 130, no. 1, pp. 155–174, 1987.

[27] C. Farley and T. Ko, "Two basic mechanisms in lizard locomotion," *J. exp. Biol*, vol. 200, pp. 2177–2188, 1997.

[28] G. A. Cavagna, N. C. Heglund, and C. R. Taylor, "Mechanical work in terrestrial locomotion: two basic mechanisms for minimizing energy expenditure," *American Journal of Physiology-Regulatory, Integrative and Comparative Physiology*, vol. 233, no. 5, pp. R243–R261, 1977.

[29] N. C. Heglund, G. A. Cavagna, and C. R. Taylor, "Energetics and mechanics of terrestrial locomotion. iii. energy changes of the centre of mass as a function of speed and body size in birds and mammals," *Journal of Experimental Biology*, vol. 97, no. 1, pp. 41–56, 1982.

[30] C. T. Farley and D. C. Morgenroth, "Leg stiffness primarily depends on ankle stiffness during human hopping," *Journal of biomechanics*, vol. 32, no. 3, pp. 267–273, 1999.